


**NOTE**  
**A SIMPLE BIJECTION BETWEEN A SUBCLASS OF 2-BINARY**  
**TREES AND TERNARY TREES**

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We consider the subclass of 2-binary trees, where

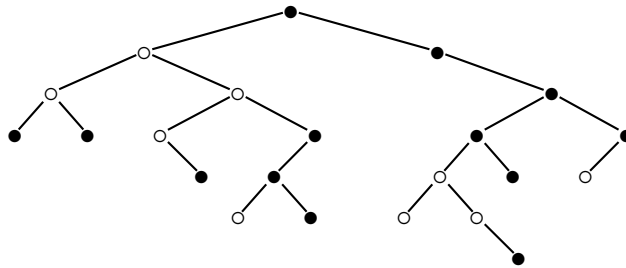
- nodes are labelled black or white,
- the root is labelled black, and
- a black node cannot have a right daughter labelled white.

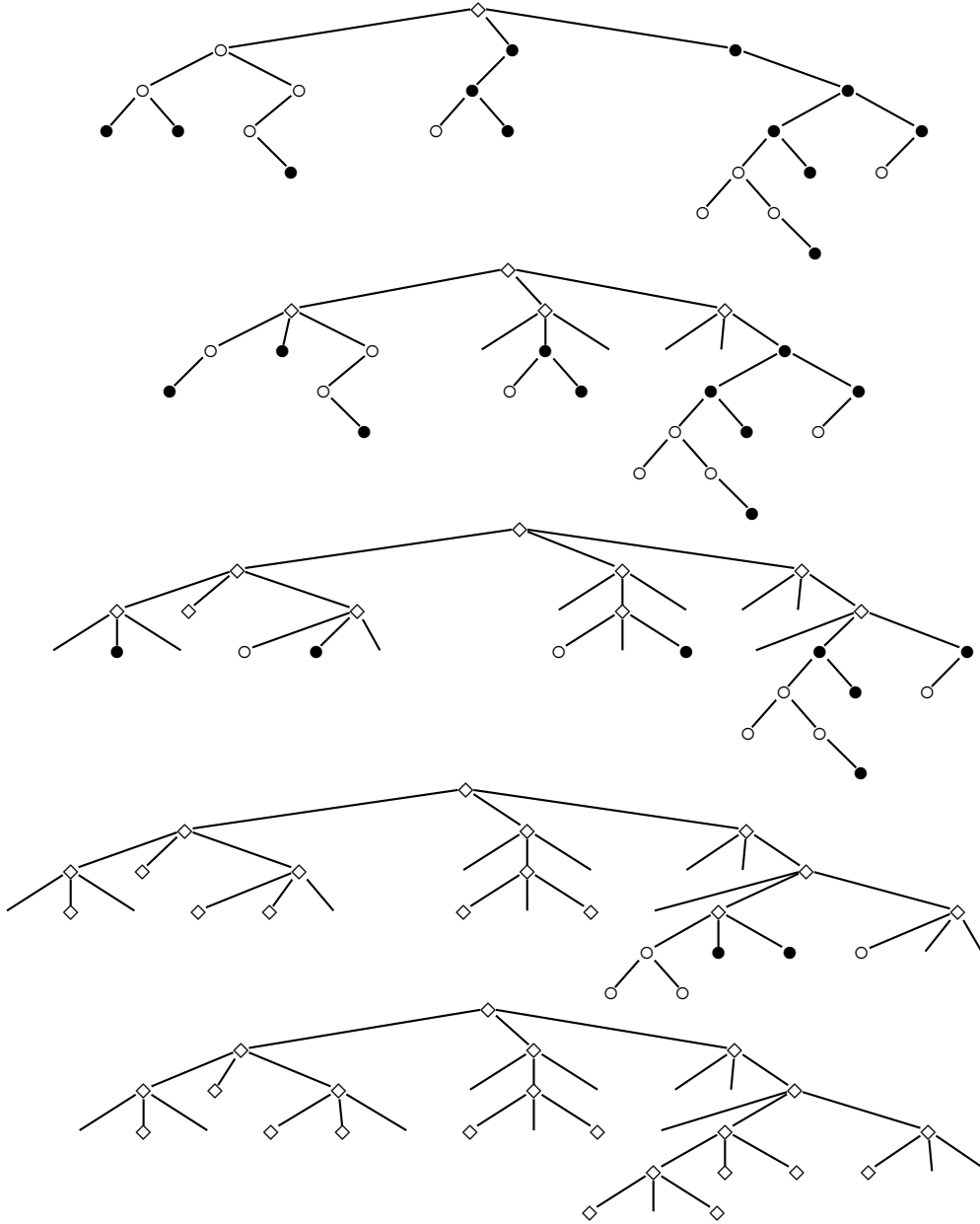
It was shown in [1] that these trees are in bijection with ternary trees (and henceforth enumerated by  $\frac{1}{2n+1} \binom{3n}{n}$ ).

In this note we provide another bijection, which is simpler. Furthermore, it preserves more of the structure of the tree. The basic idea is that, when  is forbidden, any sequence of nodes connected by right edges must have labels white ... white, followed by black ... black. We translate the restricted 2-binary tree recursively into a ternary tree from the root to the leaves, by leaving right edges intact, and connecting the translated right subtree to the root as the right(most) subtree. Now, we look locally at the root, its left subtree and the longest chain of right edges, viz.



We remove the edge between the white and black nodes (printed in boldface), thus splitting the left subtree of the root into two trees, the *white* and the *black* tree. The transformed versions of them go to the left resp. middle subtree of the root. Intuitively, the meaning of the 3 subtrees is *white*, *black*, *right*. As said already, this procedure is continued recursively. It leaves the right structure, and uses the left and middle subtrees for a canonical split of the left subtree. It is best understood by an example, given step by step. The nodes of the resulting ternary tree are displayed as diamonds, in order to distinguish them.





It is not hard to see that the procedure can be reversed.

**Acknowledment.** Thanks are due to Stephan Wagner who listened to an early presentation of this bijection.

#### REFERENCES

- [1] N.S.S. Gu, N.Y. Li, and T. Mansour. 2-Binary trees: Bijections and related issues. *Discrete Math.*, xxx:xxx–xxx, 2007.

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