MATHEMATICAL ANALYSIS OF ALGORITHMS IN EUROPE

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General. As the primary source of information on the subject, I will give the URL of the recently established homepage of "Analysis of Algorithms" that is maintained by Philippe Flajolet and myself: `http://www-rocq.inria.fr/algo/AofA/index.html`. There, one can find much more information about the subject.

The area was created by Donald Knuth in 1962 when he did his legendary analysis of hashing with linear probing.

(Average–case) Analysis of Algorithms means (at least to us!) the study of characteristic parameters of an algorithm, under a suitable probabilistic model. Not surprisingly, the emphasis is on expected values, variances, and limiting distributions. Now, discrete probabilistic models bring us into the realm of Discrete Mathematics, most prominently Combinatorics. Characteristic parameters are in most cases directly related to the running time and the amount of storage needed. Typical examples are the number of comparisons for various sorting algorithms, the size of a *trie*, etc. The topics covered are *fundamental subjects* of Computer Science, like searching, sorting, selection, organization of storage, etc. As such, they are mostly independent of implementations and programming languages and thus perhaps less ephemeral than some results in other areas of Theoretical Computer Science.

The area that is closest to us is *Analytic Combinatorics*. Generating functions play a central role, together with Combinatorial Constructions, serving as models for the Data Structures used in the Algorithms. And usually, the interest is also in the asymptotic behaviour of the parameters, especially if exact answers are too complicated or not available.

Activities of the community. Recently, the group of people interested in Analysis of Algorithms started some interesting initiatives, mostly in Europe: In 1993, 1995, 1997 there were meetings in Dagstuhl (the German Computer Science center close to Saarbrücken), solely about this subject. And after each meeting, there were special issues published about this line of research. The organizers were Philippe Flajolet from Paris, Rainer Kemp from Frankfurt and Helmut Prodinger from Vienna, with guest organizers Robert Sedgewick and Hosam Mahmoud from the U. S. The special issues in Theoretical Computer Science, Random Structures and Algorithms and Algorithmica were/are edited by Flajolet, Prodinger, and Szpankowski, in different combinations. A future cooperation with Algorithmica has been worked out with regard to future meetings, the next one being planned for Princeton, July 20-24, 1998.

Random Structures and Algorithms is a neighbouring discipline, and people in this area (mostly from Poznan, Poland) have their own journal with this name, and their own biennial series of conferences with this name (previously *Random Graphs*).

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Sites. The sites that were already mentioned are also centers of activity with Paris being the world leader. The center of gravity is INRIA with its "Algorithms Project," but several other universities and research facilities in the larger Parisean area are loosely connected. Further centers are Barcelona (where the 1999 meeting will take place), Vienna, Frankfurt, Brussels, and Florence. A prominent offspring from the French educational system is the highly promising Hsien–Kuei Hwang, who is now back in Taiwan.

The group around Philippe Flajolet concentrates on analysing particular algorithms, creating mathematical methods for the analysis, and symbolic computation. Tools like Maple and Mathematica are obviously very important for us, and there are already some efforts for an "Automatic Analysis of Algorithms." Also related is "Random Generation of Combinatorial Objects," in particular Maple's CombStruct package. Especially useful is the package Gfun by Bruno Salvy and his collaborators, that helps (among a lot of other things) to guess the general form of a sequence, from just a few initial values. Related to this is an important new feature, created by Doron Zeilberger (partially with Herbert Wilf), concerning the automatic simplification of combinatorial sums.

The Frankfurt people concentrate lately on formal languages and words and their algorithmic properties. Powerful theorems give us the average complexity of algorithms to generate words of length n lexicographically, only based on the rules of the grammar.

Activities in Vienna are of various types, perhaps with some concentration lately on exact results depending on more than one parameter, for binary search trees, (multiple) Quickselect, etc., with a combination of Maple computations and guessing (plus proving).

People in Barcelona (around Conrado Martínez) are additionally working in randomization of algorithms (notably binary search trees).

Guy Louchard from Brussels comes originally from Probability Theory. He was very successful in applying methods from the theory of *Brownian Motion* to the analysis of algorithms.

These groups are, however, not independent. Several sites are involved in the ALCOM project. And there are some formal cooperations, notably between Paris and Vienna, and Vienna and Barcelona.

Publications. There are some books that are (almost) exclusively devoted to the subject. Apart from Knuth's classic works, I mention Kemp's *Fundamentals for the Average Case Analysis of Particular Algorithms*, Mahmoud's *Evolution of Random Search Trees*, and Flajolet/Sedgewick's *An Introduction to the Analysis of Algorithms* (plus a second volume in preparation). Also Wojciech Szpankowski (originally Polish, thus European!) is currently writing *Mathematical Analysis for Algorithms on Words*.

Conclusion. This has been a report on what exists already (especially in Europe). However, there remains a lot to be done. The group is small, at least compared to some booming areas in Theoretical Computer Science. One of the reasons could be that active researchers need a solid education in Mathematics, something that usually does not happen in the contemporary Computer Science curriculum. The series of (small) research seminars should finally lead to 'real' conferences, with call–for–papers, and proceedings. My guess is that in the long run there will be a scientific journal with main emphasis on the Analysis of Algorithms. (Probably the Journal of Algorithmics was created because of a similar motive, but it drifted into another direction.) And another important point is to make it clear to the Computer Science community that it is vital to *understand* an algorithm, which means not only to know how it works but also how it *behaves*. Perhaps the present note might help in this direction.

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