Triangulations and Pseudotriangulations in Surfaces. Triangulations of surfaces with specific intrinsic metrics different from the Euclidean plane have been of relevance in connection with different mathematical problems: On surfaces with sphere topology and convex polyhedral metrics, a weighted Delaunay triangulation was instrumental in giving a constructive solution to A.D. Alexandrov's theorem on the existence of polytopes with a given metric [2]. Computational and numerical aspects of such triangulations are treated e.g. by [5, 3], and are surveyed by [9]. In the context of computational geometry, the notion of pseudo-triangulation was introduced in the 1990s, where "pseudo-triangle" refers to a simple polygon with exactly three convex vertices. The rich geometry of pseudo-triangulations has been surveyed by [8], in particular their relations to rigidity theory. Most contributions concern planar domains, but spherical pseudo-triangulations have applications in origami [10].

It is our aim to systematically explore triangulations and pseudo-triangulations in two-dimensional geometries which, except for cone points, have constant curvature. These include the surfaces of polyhedra, in particular convex ones, and the sphere. The edges occurring in triangulations are shortest geodesic paths. Our central approach will be to generalize interesting research topics on gemetric graphs in the Euclidean plane to this more general setting.

Examples of suitable questions are the equivalent of edge flips in triangulations — like flipping to the Delauny triangulation or showing the connectedness of any two triangulations via a sequence of edge flips – see [6] for an overview of existing work. Another interesting question involves (pointed) pseudo-triangulations: which properties known from the Euclidean case are still true? What are possible extensions and limitations? How can pointedness properly be defined? See e.g. [4, 7] for recent contributions.

Other research topics concern optimization (e.g., what is the complexity of computing a shortest spanning tree or a minimum weight triangulation on these surfaces?) as well as counting problems (what are lower and upper bounds for the number of different triangulations, pseudo-triangulations, and other structures on the surfaces under consideration? Existing work includes e.g. [1], we refer to [6] for an overview.

This non-exhaustive list of potential research directions shows that there are plenty of possibilities to generalize known results from plane structures to surfaces with cone points, both in an algorithmic way and in a structural/discrete way.

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